Specifics of mathematical practices in economics

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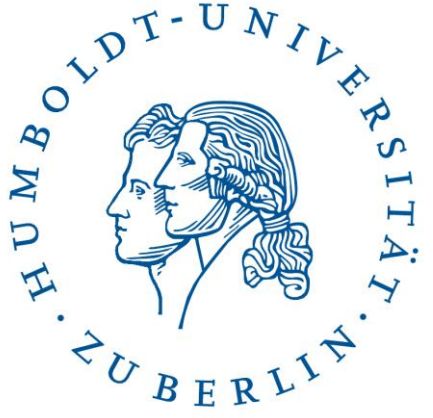

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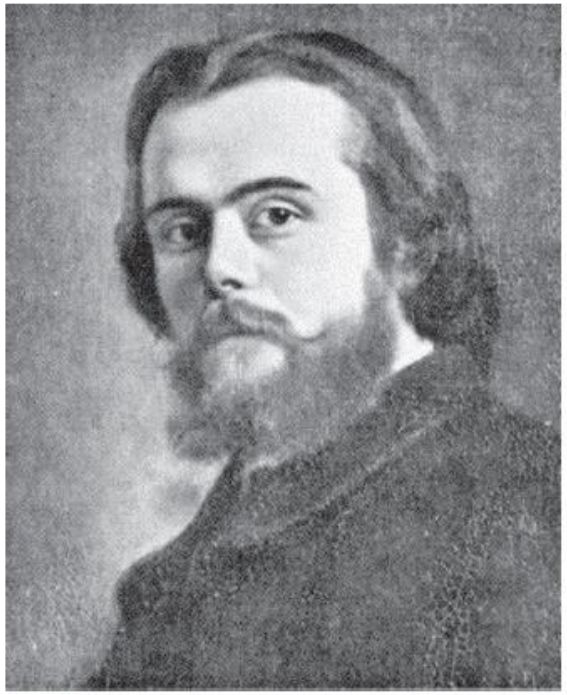


Abb. 4.3: Léon Walras
(1834-1910)

"The use of the language, methods, and principals of mathematics have the aim to make our analysis more rigorous, more exact, and more comprehensive than possible by using logic. It also contributes to drawing more exact consequences. In our rich theory, our formulas may not only be abstract, they really *need to be* in order to keep generality. Our curves and functions should be applicable to any goods that may be exchanged.

Most national economists have no idea about the mathematics, and only know the basic arithmetic operations. In their point of view, the application of mathematics in economics is limited to addition, subtraction, multiplication, and division.
They are wrong!
The application of mathematics in economics is to consider utility, output, effective demand, effect supply, price etc. as quantitates, and to reason about how these are related, and to make use of the general properties of these functions for investigating economic phenomena."

(Reiß, 2007, p. 185, citation of Walras according to Jaffé, 1971)

But what mathematics is needed?

Mathematics plays a crucial role in economics, as it provides the tools and techniques necessary for modeling, analyzing, and understanding economic phenomena. […]

Several mathematical concepts are involved in studying economics. These include calculus, linear algebra, statistics, optimization, game theory, and differential equations.

ics-What-concepts-

Is it appropriate to just teach the students these topics as in the usual mathematics courses (maybe in a less abstract way)?

Outline:

1. Contextual interpretations of mathematical concepts
2. Specific mathematical concepts
3. Mathematical reasoning in economics
4. Modelling
5. Summary

Economic interpretation of the derivative:

Definition of marginal cost in a standard economics textbook:

(Wöhe und Döring, 2010, p. 309-310)

„The marginal cost C' are the cost of the last unit. Hence, you can determine the marginal cost of the 33rd unit by subtracting the total cost of 32 units from the total cost of 33 units. The marginal cost represents the slope of the original cost function. It can be determined by taking the derivative of the original cost function:

$$C' = \frac{dC}{dm}“$$

Economic interpretation of the derivative:

Definition in mathematics:		**Interpretation in economics:**
If C: $[0;\infty[\longrightarrow [0;\infty[$ is a cost function, marginal cost is defined as: $\mathrm{MC}(x) := C'(x) = \lim_{h \to 0} \frac{\mathrm{C}(x+h) - C(x)}{h}$ (if the limit exists)		„Marginal cost is the additional cost that will arise if the output is increased by one unit.“ (Pindyck and Rubinfeld (2008), p.301) $MC(x) = C(x+1) - C(x)$

Connection between marginal cost as derivative and marginal cost as the additional cost:

Approximation formula:

$$C(x+h)-C(x)\approx C'(x)\cdot h \quad \text{for} \quad h\approx 0.$$

h=1 is small in economics!

In my PhD Thesis, I investigated to what extent students can make this connection and make sense of this economic interpretation from a cognitive perspective.
(Feudel & Biehler, 2022)

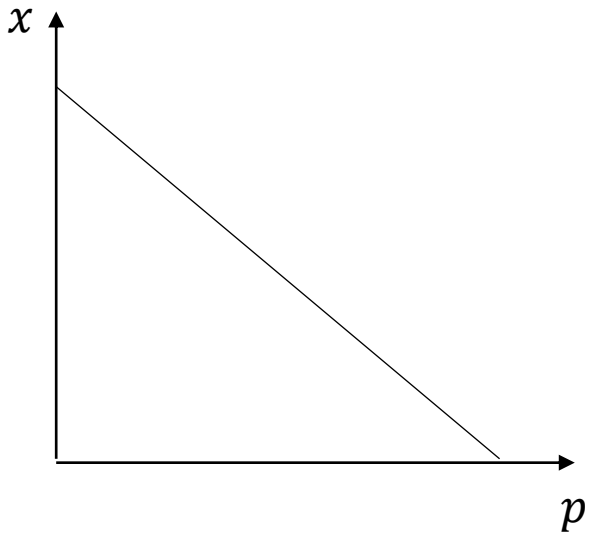
2. Specific mathematical concepts

The problem of sensitivity of demand in dependence on the price:

Question:

How strong does a change in the price affect a monopolist's revenue?

The price decreases as the demand decreases!



Slope/Derivative as measure for sensitivity?

The problem of sensivity of demand in dependence on the price:

It seems more appropriate to consider relative changes: $\frac{\frac{\Delta x}{x}}{\frac{\Delta p}{p}}$

If $\Delta p \to 0$, we get: $\varepsilon := \frac{dx}{dp} \cdot \frac{p}{x}$

Concept “Price elasticity of demand”

Effect of price changes on revenue: $R'(x) = p(x) \cdot \left(1 + \frac{1}{\varepsilon(x)}\right)$ (Amoroso-Robinson Relation)

Students understanding of elasticity:

Economics students seem to have a very weak understanding of the meaning of elasticity, and – if at all – mostly just remember the calculation formula

(Mkhatshwa, 2024)

3. Mathematical reasoning in economics

The problem of profit maximization in dependance of the input:

(Varian, 2014)

Given:

- Firm that produces one good out of two inputs, production process described via a production function $f(x_1, x_2)$
- Input prices w_1, w_2
- Product selling price p

To find:

- Amount of goods for maximal profit (for simplicity if good 2 is fixed at a level $\overline{x_2}$)

Solution:

Solve $p \cdot MP_1(x_1, \overline{x_2}) = w_1$

Context-based derivation of the criterion:

$$pMP_1(x_1^*, \overline{x_2}) = w_1$$

Change in output

Change in revenue

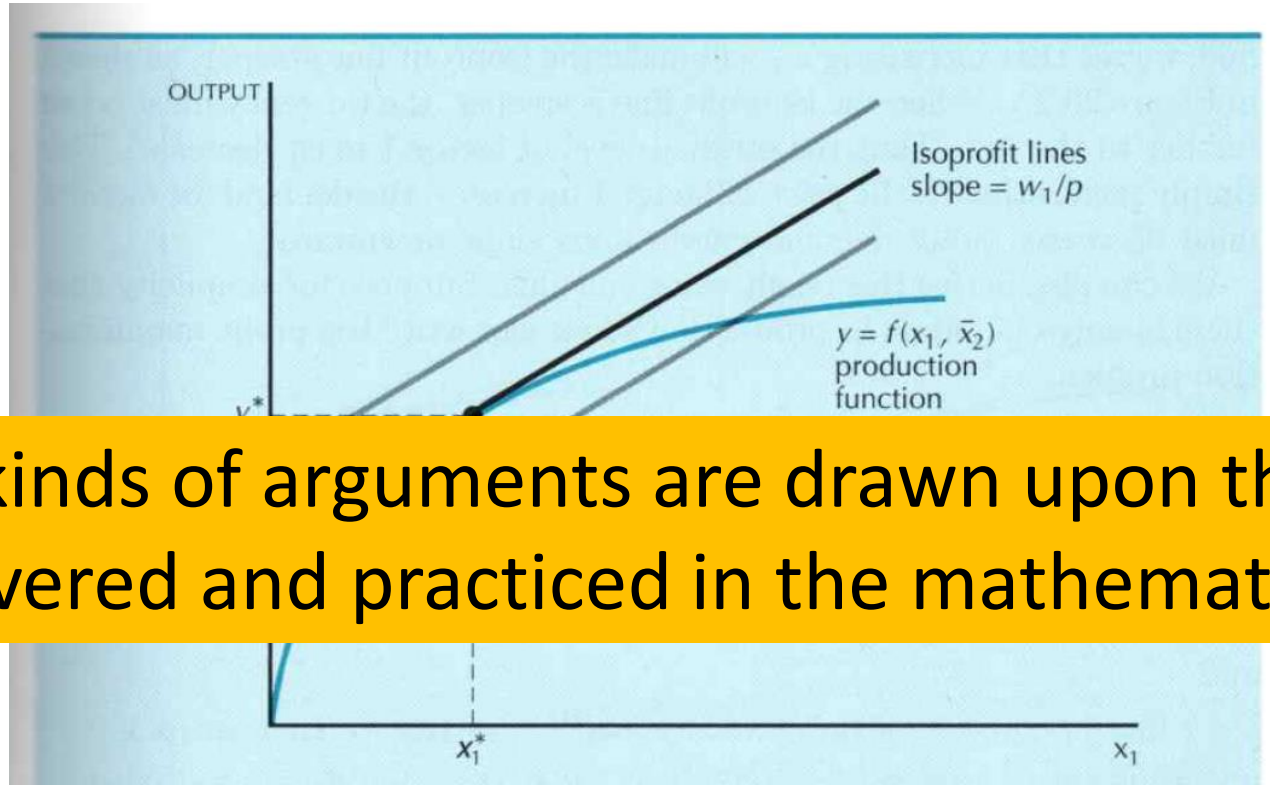
In order to understand this rule, think about the decision to employ a little more of factor 1. As you add a little more of it, δx_1, you produce $\delta y = MP_1 \delta x_1$ more output that is worth $pMP_1 \delta x_1$. But this marginal output costs $w_1 \delta x_1$ to produce. If the value of marginal product exceeds its cost, then profits can be increased by *increasing* input 1. If the value of marginal product is less than its cost, then profits can be increased by *decreasing* the level of input 1.

Change in costs

(Varian, 2014, p. 372)

Graphical derivation of the criterion:

$$pMP_1(x_1^*, \overline{x_2}) = w_1$$



Different kinds of arguments are drawn upon than the ones usually covered and practiced in the mathematics courses!

(Varian, 2014, p. 372)

At the optimal output the slope of the isoprofit line needs to be equal to the slope of f !

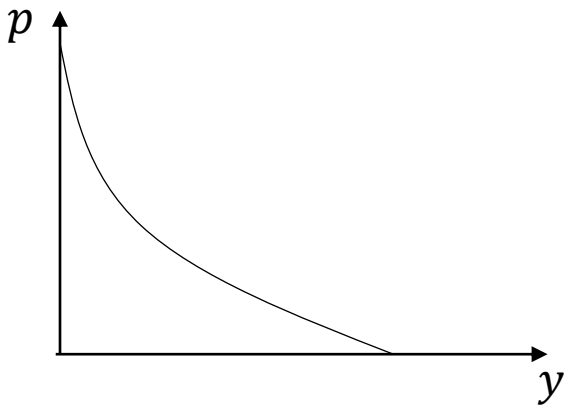
$$\Rightarrow \quad \underline{MP_1(x_1, \overline{x_2}) = \frac{w_1}{p}}$$

4. Modelling

What price should a monopolist set for maximal profit?

Assumption:
The demand only depends on the price, and the demand decreases as the price increases.

The relationship can be modelled with a decreasing function (the demand function)



What price should a monopolist set for maximal profit?

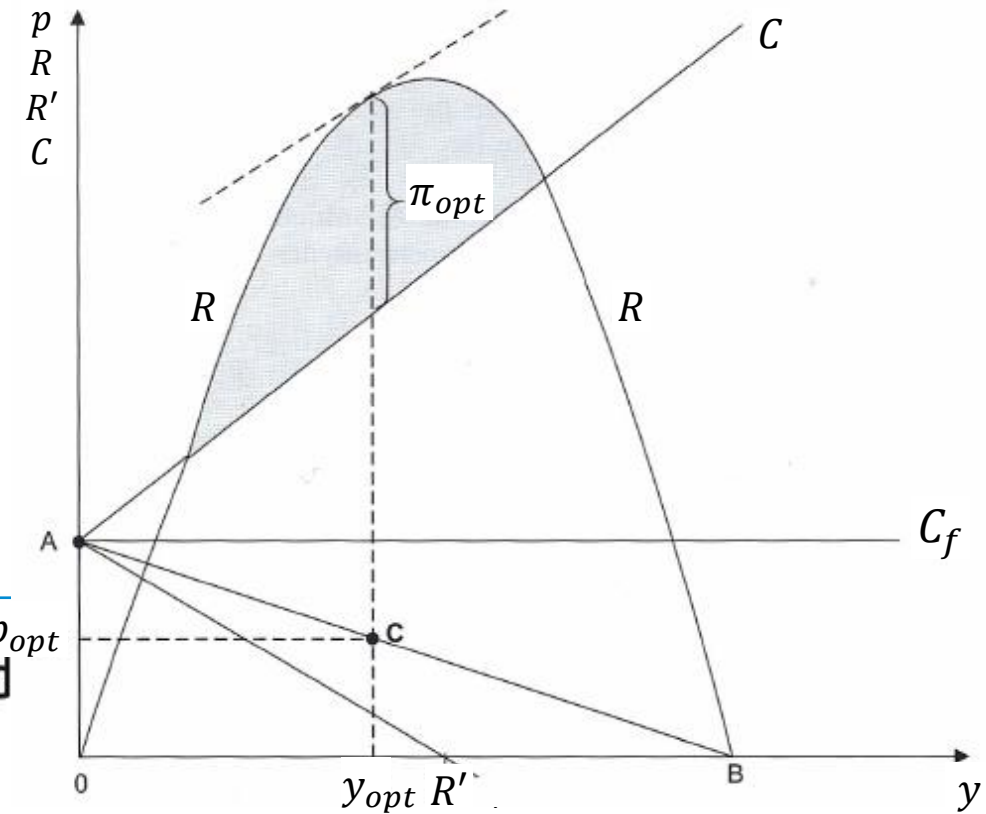
What are price and output for maximal profit?

$$\pi(y) = R(y) - C(y)$$

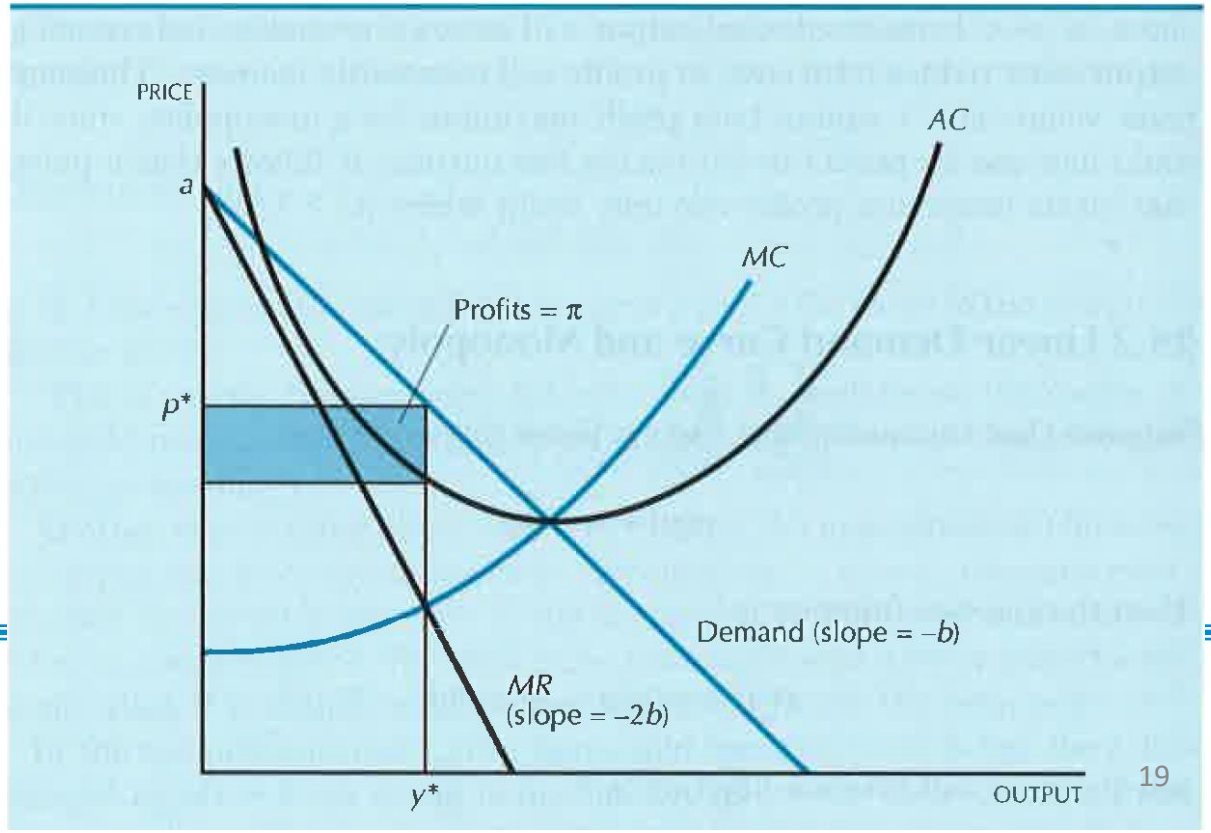
At maximal profit, we have $\pi'(y) = 0 \;\Rightarrow\; R'(y) = C'(y)$ or $MC(y) = MR(y)$

Examples with linear demand curves $p(y) = a - 2by$

Wöhe & Döring (2013, p.423)



Varian (2014, p.464)



cepts

What price should a monopolist set for maximal profit?

The aim of such a modelling process is not to calculate a specific number, but to derive relationships between economic quantities involved in the problem under certain economic conditions and assumptions.

The results are usually not checked with data, as empirical testing cannot be easily realized in economics.

Suggestion of a description of modelling in economics by Rainer Voßkamp (2023):

Model 1

An adequate model M is defined due to a set of assumption (definitions, hypotheses) $A.1$ to $A.n$:

$$M = \{A.1, \ldots, A.n\}$$

Theorem 1

A theorem is formulated related to statement of X

Proof 1

Proof of the theorem using $A.1$ to $A.n$.

5. Summary

Summary:

- Mathematics in economics involves much more than simple calculation procedures.
- Contextual interpretations play an important role, and these might not coincide to how the concept is usually taught in mathematics.
- There are specific concepts that are relevant for economics but only shortly covered in standard first-year mathematics courses.
- Mathematical processes like reasoning or modelling differ from related processes in mathematics as a scientific discipline.

It is therefore important to develop mathematics courses that take these special features into account.

Literature:

- Feudel, F., & Biehler, R. (2022). Students' understanding of the economic interpretation of the derivative in the context of marginal cost. *International Journal of Research in Undergraduate Mathematics Education*, *8*(3), 437-468.
- Jaffé, W. (1971). Marie Esprit Léon Walras (1842-1924). In: Recktenwald (pp. 344–357)
- Mkhatshwa, Thembinkosi P. (2024). An investigation of business calculus students' covariational reasoning, procedural knowledge, and conceptual knowledge in the context of price elasticity of demand. *Teaching Mathematics and its Applications: An international journal of the IMA.*
- Pindyck, R. S., & Rubinfeld, D. L. (2008). *Mikroökonomie* [Microeconomics]. Pearson Deutschland GmbH.
- Reiß, W. (2007). *Mikroökonomische Theorie: historisch fundierte Einführung* [Microeconomic theory: a historically-grounded introduction]. Walter de Gruyter GmbH & Co KG.
- Smith, A. (1023): *Sammlung sozialwissenschaftlicher Meister. Bd. 11: Eine Untersuchung über Natur und Wesen des Volkswohlstandes* [Collection of great social scientists, No. 11: An investigation on the nature of social welfare]. Fischer, 1923. – Unter Zugrundelegung der Übersetzung Max Stirners, aus dem englischen Original nach d. Ausg. letzter Hand, 4. Aufl. 1786, ins Deutsche übertragen von Prof. Dr. Ernst Grünfeld u. eingeleitet von Prof. Dr. Heinrich
- Varian, H. R. (2014). *Intermediate microeconomics with calculus*. W.W. Norton & Co.
- Voßkamp, R. (2023). Calculus in mathematics for economists. In T. Dreyfus, A. S. González-Martín, E. Nardi, J. Monaghan, & P. W. Thompson (Eds.), *The Learning and Teaching of Calculus Across Disciplines – Proceedings of the Second Calculus Conference* (pp. 51-68). MatRIC.
- Wöhe, G., & Döring, U. (2013). *Einführung in die allgemeine Betriebswirtschaftslehre* [Introduction to general business administration]. Vahlen.

Thank you very much for your attention.